Planet Habitability in a Binary Star System

James Kath

Evanston Township High School

1 Purpose of Research

It has long been thought that planets form mainly in single star systems similar to our own, but this assumption has recently been challenged. Data collected over the past decade from the Hubble Space Telescope suggests the existence of a Jovian-mass planet orbiting the millisecond pulsar PSR B1620-26 and its companion white dwarf [12]. This is the first evidence of an extrasolar planet orbiting a binary star system.

Observational evidence suggests that at least half of all visible stars make up binary star systems [2]. If planets can form just as readily in binary systems, then the existence of life in these systems is a possibility that cannot be ruled out. However, the companion star's radiation and perturbing influence may alter the climate enough to make a candidate planet unsuitable, making the question of habitability a much more complex and interesting problem.

The purpose of this study is to address this problem by finding a habitable range of orbital size and star separation distance. A review of the literature did not find any previous attempts to do this. Since the purpose of this study is not to define life or postulate on all the factors involved in the formation of life, I will use an Earth-like climate as a definition of habitability, namely a temperature range that would support the existence of liquid water.

2 Background

2.1 Orbital Motion

To simulate a planet orbiting within a binary star system, a three-body model must be considered in which each body gravitationally interacts with the other two. The

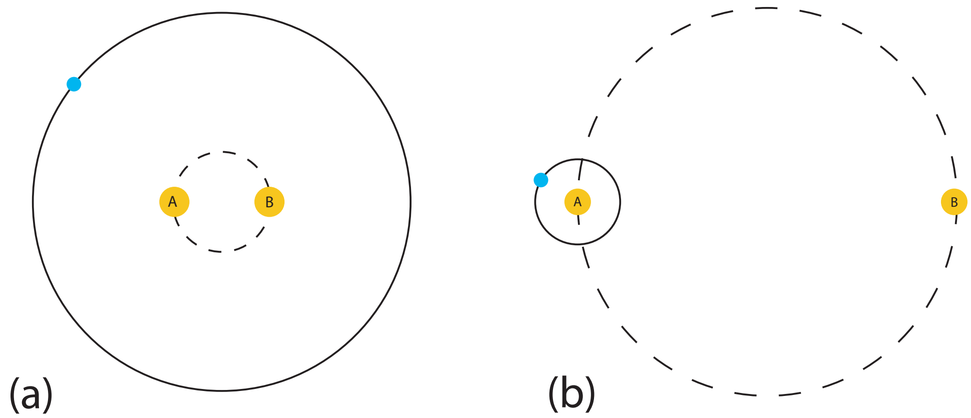

Figure 1: A sample external (a) and internal (b) orbit

equations used in determining gravitation and orbital motion are Newton's second law of motion:

$$\vec{F} = m\vec{a} \tag{1}$$

and Newton's law of gravitation:

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\,\hat{r}\,. \tag{2}$$

Peterson states that orbital motion of a Newtonian system consisting of more than two bodies cannot be predicted exactly, and must be solved numerically [7].

There are two general types of possible orbits within a binary system – internal and external. A planet in an external orbit has a large mean radial distance relative to the stars' separation distance and orbits around both stars, whereas a planet in an internal orbit has a small relative mean radial distance, and orbits mainly around a single star (see Figure 1). For simplicity, this paper deals only with internal orbits.

This study deals only with resonant periodic orbits, or RPOs. A planet that exhibits resonance in a binary star system would have an orbital period that is either a fraction (for internal orbits) or a multiple (for external orbits) of the stars' orbital period.

Haghighipour states that such resonances create regions of stability in the orbital phase space [3]. For example, the orbits of Neptune and Pluto around the Sun are in stable resonance [7]. Stability is a crucial factor in the formation of life, since the time scales involved measure in billions of years and an unstable orbit would most likely spiral into or away from a star in a much shorter period.

2.2 Energy Budget

A simple energy budget was used to find the planetary temperature, T_p, in terms of position and stellar properties and to determine a habitable range. The simplest model to determine temperature is the Stefan-Boltzmann Law:

$$\frac{I_0}{4\pi d^2} = \sigma T_p^4 \,. \tag{3}$$

This model treats the planet as a blackbody, an object that absorbs all incident radiation and re-emits it. The left side of the equation defines the flux density of solar radiation at a distance d, where I_0 is the star's luminosity constant (3.827×10^{26} J/s for the Sun) and the factor of 4π accounts for the fact that radiation ideally spreads spherically. The constant of proportionality σ is known as the Stefan-Boltzmann constant and has a value of $5.67 \times 10^{-8}\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{K}^4)$.

This model can be improved upon by adding two new terms to the Stefan-Botzmann law:

$$\frac{I_0}{16\pi d^2}(1-\alpha) = \sigma T_p^4 \,. \tag{4}$$

The constant α is the *planetary albedo*, or the percentage of incident radiation that is immediately reflected off a particular planet. The value of albedo for the Earth is about 0.3 [7]. The additional factor of four in the flux density term estimates that solar

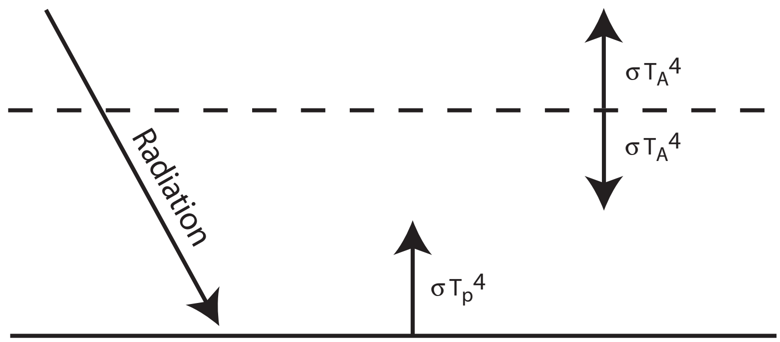

Figure 2: A simple model of the greenhouse effect

flux hits the Earth with a cross section of a circle, with area πr_p^2, and is reemitted as a sphere with area $4\pi r_p^2$, where r_p is the planet's radius [4]. The final addition to the energy budget equation is a simple model of the greenhouse effect [8]:

$$\frac{I_0}{16\pi d^2}(1-\alpha) + \sigma T_A^4 = \sigma T_p^4 \quad \text{and} \quad \sigma T_p^4 = 2\sigma T_A^4\,, \tag{5}$$

where T_A is the atmospheric temperature, and σT_A^4 is the energy flux radiated by the atmosphere.

This model treats the planet's atmosphere as a blackbody itself, which does not inhibit the transmission of solar radiation to the surface but absorbs and remits the planetary emissions (see Figure 2). Using this fact, σT_A^4 can be solved in terms of σT_p^4, reducing the equation to one variable. Thus, the final equation for temperature in term of distance is:

$$T_p = \sqrt[4]{\frac{I_0}{8\pi\sigma d^2}(1-\alpha)}\,. \tag{6}$$

3 Methods

In this study, a variety of approximation techniques were used to simulate the possible orbits and climates of an Earth-like planet in a binary system and to determine habitability. These techniques include numerical approximations to simulate orbital motion, the Newton method to obtain closed orbits from near-closed initial orbits, and a simple energy budget to calculate temperature. All computer simulations were originally programmed and run using MATLAB 6.1 [5] under Windows 98 on an 800 MHz Pentium III processor.

3.1 Orbital Motion

As stated before, modeling the motion of objects influenced by the gravity of two or more bodies is a problem that cannot be solved exactly in closed form. Instead, numerical integration techniques must be used that break the bodies' motion into discrete steps.

This study used a well-known numerical integration technique called the fourth-order Runge-Kutta method, which was used in this study to program the ordinary differential equation solver. The fourth-order Runge-Kutta method is a fourth-order explicit that takes an initial Euler approximation for the change in position and velocity and extrapolates throughout the numerical step to obtain three other correcting approximations [1]. The final values of velocity and acceleration that are used to advance the step are obtained by a weighted average of all four approximated terms:

$$k = \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), \tag{7}$$

where k_1 is the initial Euler approximation and k_2, k_3, and k_4 are the correcting terms.

This Runge-Kutta method creates a simulation in which error varies inversely with the fourth power of the number of steps. Although there are more accurate numerical integration techniques, the power and simplicity of the fourth-order Runge-Kutta method makes its use practical. In this study, 10,000 steps were used to model each orbit, which translates to step sizes varying between 11.91 seconds and 4.69×10^4 seconds (13.01 hr.), depending on the orbital period.

To further simplify the model used in this study, a restricted three-body model was used. In the restricted model, the gravitational pull of the planet is considered negligible [6]. This is a reasonable simplification since the ratio of the mass of an Earth-like planet to the mass of either star is approximately 3×10^{-6}. Additionally, this study deals with two dimensions, a justifiable assumption since solar systems tend to form on planes.

Finally, a rotating coordinate system was used. As the name suggests, this coordinate system removes the rotation of the stars, which nullifies the necessity of modeling their orbits as well. This system also requires center-of-mass coordinates for the two stars, and assumes circular orbits [6]. In the rotating frame, equations for acceleration ($\vec{a}_r$) are now altered to include the rotation of the coordinate system:

$$\vec{a}_r = \vec{a} - 2\vec{\omega} \times \vec{v} - \vec{\omega} \times (\vec{\omega} \times \vec{r}), \tag{8}$$

where $\vec{\omega}$ is the angular velocity, $\vec{v}$ is the velocity, and $\vec{r}$ is the radial distance. The term $\vec{\omega} \times (\vec{\omega} \times \vec{r})$ in the new acceleration equation accounts for the false centrifugal force and the term $2\vec{\omega} \times \vec{v}$ for the coriolis force, a rotational force that is partly responsible for weather patterns on the Earth.

3.2 Newton Method

To obtain RPOs, this study uses a multivariable form of the Newton method, an iterative calculus technique commonly used to estimate roots of polynomial functions.

To use the Newton method, the difference between the initial and final x-position (x), y-position (y), x-velocity (u), and y-velocity (v) of an orbit is treated as a vector function, $\vec{f}(\vec{x})$, dependent on $\vec{x}$, which is the vector of initial values of the four variables. Given the initial conditions of an RPO, this function will, by definition, equal zero [10].

This method starts with an almost periodic orbit, which can be obtained by "hunting" with the orbital simulation program. For such an orbit, $\vec{f}(\vec{x})$ will be non-zero, but there exists a vector $\Delta\vec{x}$ that, when added to the vector of initial conditions, will create an RPO:

$$\vec{f}(\vec{x} + \Delta\vec{x}) = 0\,. \tag{9}$$

The vector $\Delta\vec{x}$ cannot be solved for exactly, so the Newton method uses an approximation of the above function to estimate $\Delta\vec{x}$:

$$\vec{f}(\vec{x} + \Delta\vec{x}) \approx \vec{f}(\vec{x}) + \frac{\partial \vec{f}}{\partial \vec{x}}(\vec{x}) \cdot \Delta\vec{x} = 0\,. \tag{10}$$

This matrix equation must be solved each time the Newton method is iterated; both the final value of $\vec{f}(\vec{x})$ after the orbital period, which can be obtained from the Runge-Kutta simulation, and the Jacobian, $\frac{\partial \vec{f}}{\partial \vec{x}}(\vec{x})$, are needed. The latter matrix also cannot be determined exactly and must be numerically integrated along with the position and velocity in the orbital simulation, using the Runge-Kutta method.

Several simplifying assumptions make the calculation of the two matrices simpler. First, conservation of energy allows one of the two position and two velocity variables to be determined in terms of the other three, so the differentiation of only the variables x, y, and v is needed. Also, to guarantee that there is only one set of initial conditions for each RPO, each iteration will be required to change just the initial values of y_0, u_0, and v_0, leaving x_0 and the period constant, so that only the differentiation of $\vec{f}(\vec{x})$ with respect to only y_0, u_0, and v_0 is needed.

3.3 Stability

RPOs used in this study must also be stable to be considered candidates for habitability. A stable orbit is defined as one that, when perturbed, will not diverge exponentially. To test this, the previously used Jacobian can be extended to include the differentiation of u, and differentiation with respect to the initial x-position, x_0, forming a complete monodromy matrix M:

$$M = \begin{bmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial y_0} & \frac{\partial x}{\partial u_0} & \frac{\partial x}{\partial v_0} \\ \frac{\partial y}{\partial x_0} & \frac{\partial y}{\partial y_0} & \frac{\partial y}{\partial u_0} & \frac{\partial y}{\partial v_0} \\ \frac{\partial u}{\partial x_0} & \frac{\partial u}{\partial y_0} & \frac{\partial u}{\partial u_0} & \frac{\partial u}{\partial v_0} \\ \frac{\partial v}{\partial x_0} & \frac{\partial v}{\partial y_0} & \frac{\partial v}{\partial u_0} & \frac{\partial v}{\partial v_0} \end{bmatrix}. \tag{11}$$

If the orbit is stable, the four-by-four monodromy matrix will have four eigenvalues, with two real values equaling one, and two complex values with magnitudes of one [3]. Since the monodromy matrix solved in this program is an estimation, a convergence test can be used to test the stability for each orbit.

3.4 Parameters

In a study such as this, there are many variables. As stated before, only internal orbits will be considered. Also, for simplification, only one binary star system will be considered, in which both stars are equal in mass and luminosity to our Sun. This choice is arbitrary. In addition, the planet modeled in this study has an Earth-like mass, albedo, and atmosphere. This was chosen because currently, the only known habitable planet in the universe has these characteristics.

4 Results

Forty RPOs were solved for using the Newton method and their global climates analyzed (see Figures 3 and 4). To obtain a representative group of orbits, four sets of ten orbits were found. For each set, the two stars were given a different separation distance D_S, varying between 1 AU (Astronomical Unit, 1.496×10^{11} meters) and 13 AUs. Within each set, the planet's mean radial distance, r_{avg}, was varied.

The orbits studied were close to circular. Using a plot of distance versus time (see Figure 5), the greatest deviation of distance from r_{avg} was found for each orbit. Typically, the larger the ratio of r_{avg} to D_S, the larger the deviation. For RPOs used in this study, the greatest deviation was 12.02%, although most were under 10%. At high $r_{\text{avg}} : D_S$ ratios, orbits tend to be even less circular and are more likely to be unstable.

As Figure 4 shows, a temperature plot for this type of internal orbit starts at some temperature, spikes, and returns to the initial temperature. This is expected, as the combined incident radiation absorbed by the planet increases as its orbit takes it closer to the second star.

Figure 4 plots the amplitude (difference in maximum and minimum values) in the temperature graph, A_{temp}, for each set of orbits as a function of the mean radial distance r_{avg} and fits it to a power function. Scaling the units to one AU enables each set to be compared to the others. A_{temp} is scaled by multiplying by $\sqrt{D_S}$ and r_{avg} is scaled by dividing by D_S, where r_{avg} and D_S are measured in AUs.

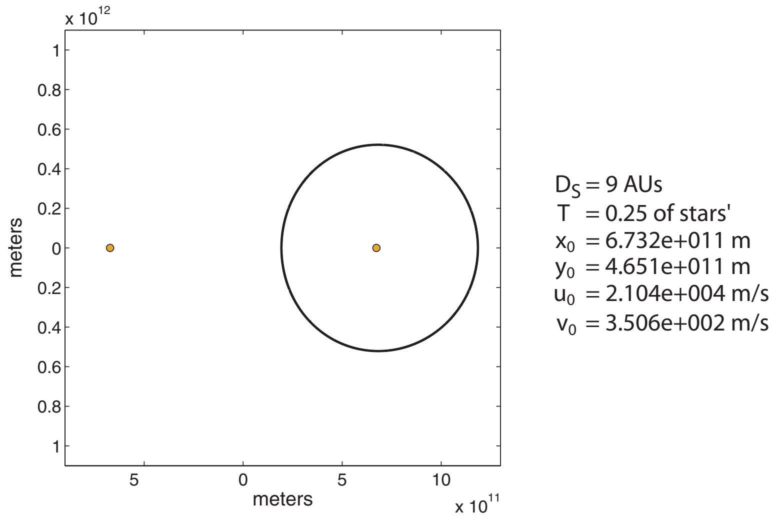

Figure 3: A sample orbit with initial conditions and period

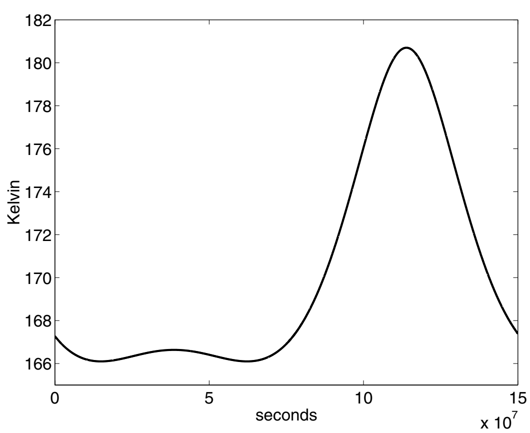

Figure 4: Temperature versus time for one period

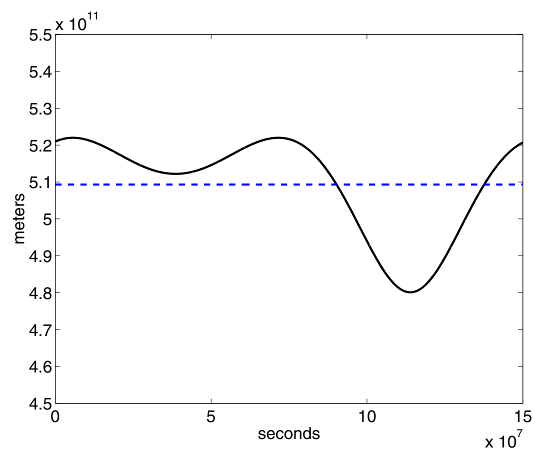

Figure 5: Distance (solid line) and r_{avg} (dotted line) from primary star versus time for one orbit

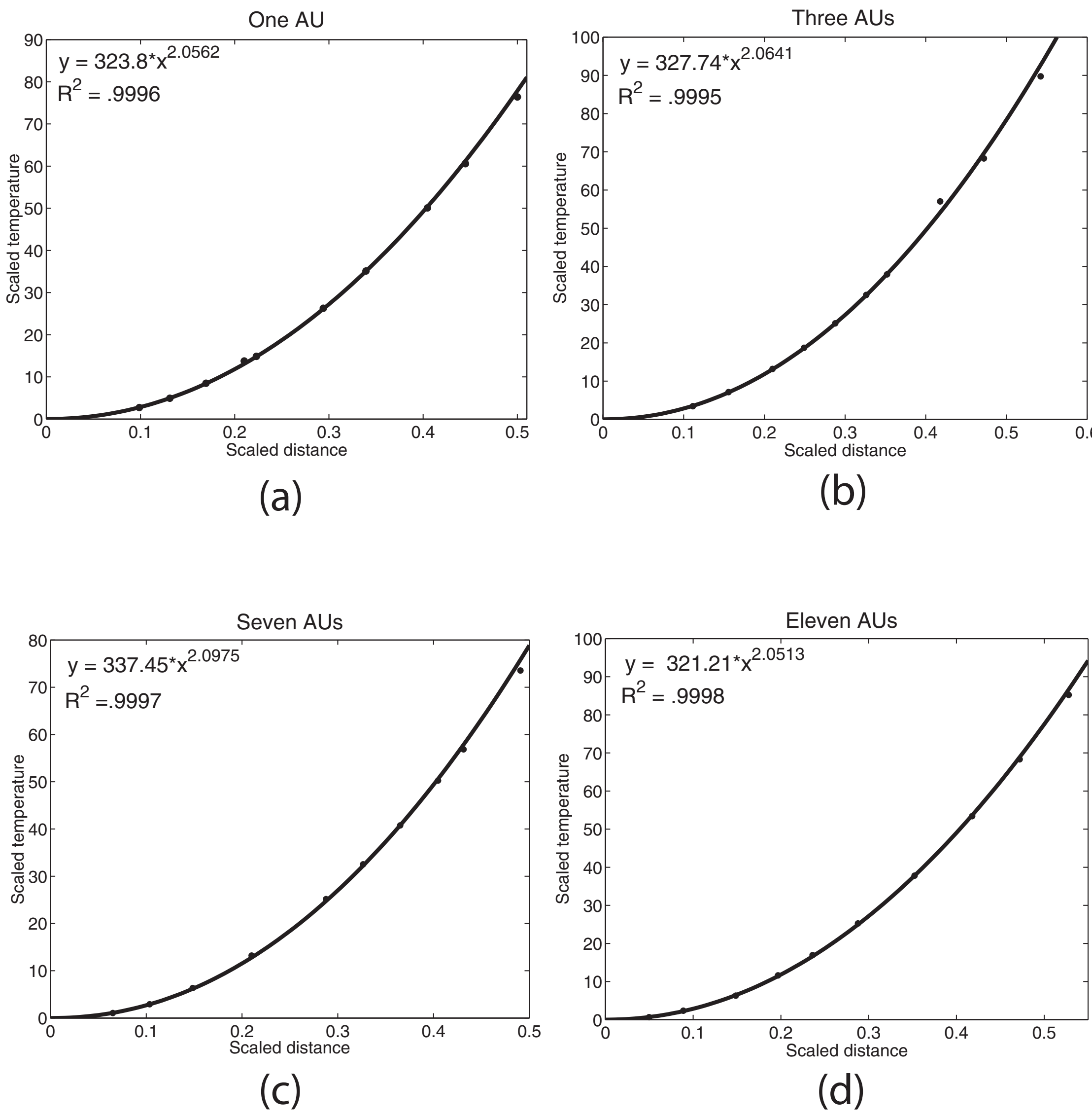

Figure 6: Scaled A_{temp} as a function of scaled r_{avg}, for four values of D_S

If all forty points are fit to a single power curve and returned to dimensional units, the relationship is:

$$A_{temp} = 327.82 \cdot \frac{r_{avg}^{2.068}}{D_S^{2.568}} , \tag{12}$$

where r_{avg} and D_S are measured in AUs. It is possible that the exponents should ideally be 2.0 and 2.5, and that the error is a result of the deviation of orbital distance from the mean value.

5 Discussion

5.1 Habitability

Previously, temperature models have been used to solve for habitable zones of single star systems, usually defined as the range of distances in which water can exist in liquid form. Using the energy budget model outlined in Equation 6, the habitable zone of the Sol system was found to range between 0.66 and 1.23 AUs from the Sun (see Figure 7).

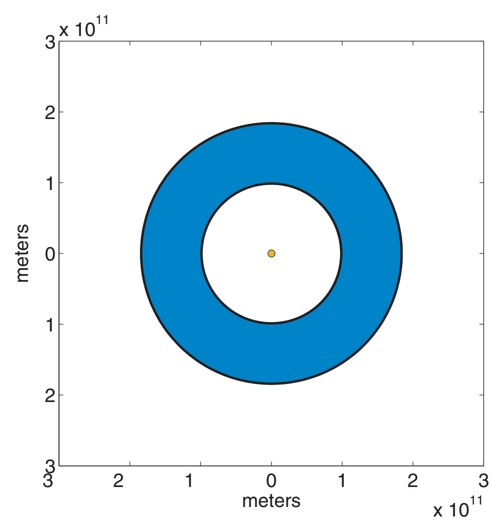

Figure 7: Habitable range of the Sun (shaded blue)

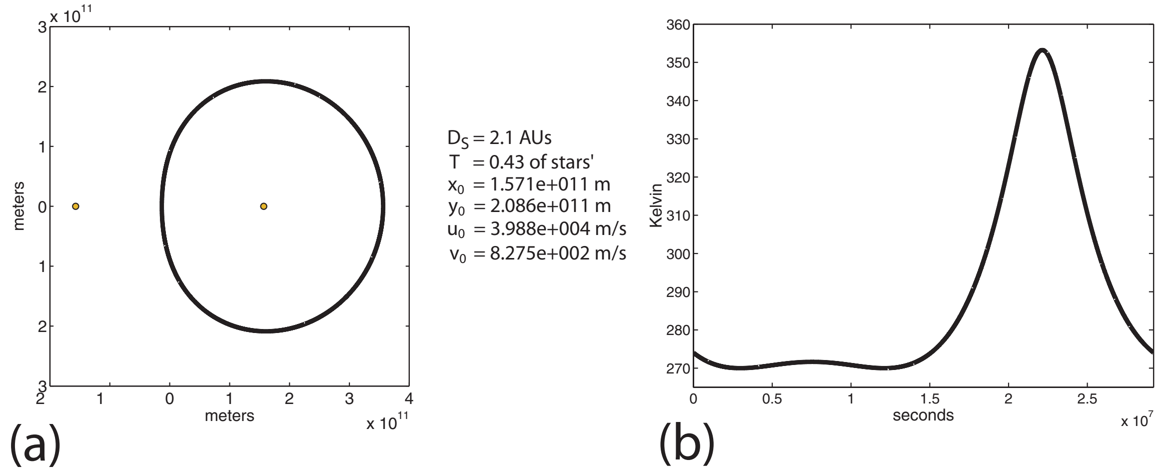

Figure 8: The limiting case, solved for using Equations 6 and 11

The existence of a second, identical star influencing the energy budget of the planet complicates the problem of defining a habitable zone. Using the energy budget model in Equation 6 and Equation 12, a system of equations can be set up that solves for a limiting case of habitability in a binary star system that has two characteristics. The first is that the orbit has a minimum temperature just above the freezing point of water (273° K) and the second is that the planet has a spike in temperature no more than 100°, leaving the maximum temperature below the boiling point (373° K).

It has been determined that such a limiting case exists if r_{avg} equals 1.4 AUs and D_S equals 2.1 AUs (see Figure 8). If the value of D_S is any smaller, the value of A_{temp}, the temperature spike, would be greater than 100°, and the planet's temperature would not remain within the boiling and freezing points.

The possibility of the existence of habitable planets increases as the value of D_S increases. At a value of 2.1 AUs, there is theoretically only one stable orbit that remains within the habitable temperature range. As the distance increases, so does the range of habitable values of r_{avg}. As D_S approaches infinity, the effects of the companion star become negligible, and the habitable range of r_{avg} quickly approaches the range of 0.66 to 1.23 AUs, the range solved for a single star.

The results of this study can be applied to observed binary systems. The neighboring α Centauri system has been considered as a potential candidate for the formation of planets and even life. The two stars in this system, α Centauri A and B, are both close in luminosity and mass to the Sun and have a value of D_S varying in between 11.4 and 36 AUs [11]. Since the α Centauri system consists of two Sun-like stars, and always has a value of D_S significantly greater than 2.1 AUs, this study concludes that it is a candidate for habitable systems if planets have formed in suitable orbits.

5.2 Temperature Models

This study used a simplified greenhouse effect to model climate. Although it is a reasonable model for temperature ranges close to that of our planet, it becomes increasingly inaccurate as the temperatures deviate from this range [9]. This study could be improved by replacing this model with one that uses experimental data to estimate the effects of CO_2 and water vapor on climate.

In addition, a simple model of ocean mixing can be used to estimate temperature:

$$\frac{dT}{dt} = \frac{S(t) - \sigma T_p^4}{\rho_w \, c_p \, h}, \tag{13}$$

where $S(t)$ is incident energy flux as a function of time, σT_p^4 represents planetary emissions, ρ_w is the density of water, c_p is the specific heat of water at constant pressure, and h is the height of the ocean's mixing layer, about 50 meters on average for the Earth [9].

The addition of ocean mixing to an energy budget tends to average temperature

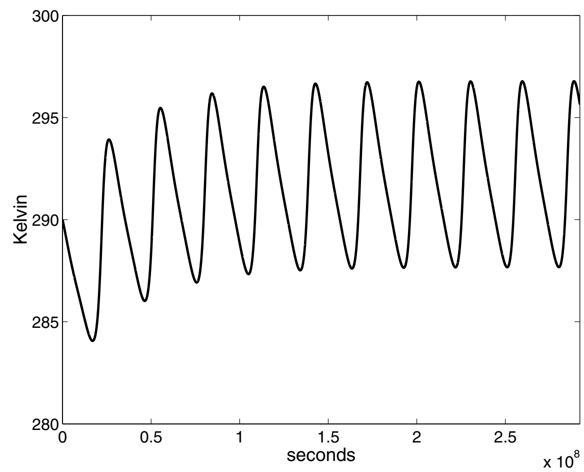

Figure 9: Temperature plot of the limiting case with a deep ocean, for ten periods

variations over a long period of time and can drastically change planetary conditions as Figure 9 shows. This figure shows the temperature plot for ten periods of the limiting case modeled in Figure 8, given an ocean with a mixing layer of 50 meters and an initial temperature of 290° Kelvin. The planet's climate approaches a sinusoidal equilibrium with the varying levels of radiation, with peaks at roughly 297° and troughs at roughly 288° Kelvin.

This model shows that a planet with a sufficiently deep ocean will be able to stand greater variances of radiation and have a larger habitability zone due to the tendency of the mixing layer in a planet's ocean to average temperature variations.

5.3 Further Research

There are many avenues for further research. As demonstrated above, the addition of an ocean to the energy budget model greatly changes planetary conditions. Studies could be done to compare different energy budget models and the effect they have on habitability zones in both single and binary star systems.

As stated before, the choice of a binary system in which both stars are identical in mass and luminosity to the Sun was arbitrary. Further studies could consider systems in which these values are not equal. Since there is a mass-luminosity relationship for main sequence stars, this reduces the problem to two variables, one for each star. These variables could even be chosen to specifically model observed systems, such as α Centauri.

In addition, external orbits could be considered. This study concludes that there is a minimum star separation distance D_S required for the existence of habitable internal orbits, and solves for one case. Conversely, it is likely that there is a maximum value of D_S for which the existence of habitable external orbits in a binary system is possible. Finally, even more exotic types of orbits could be considered. If the mass ratio of a star to its companion is large enough, objects can stably orbit in or around the system's L4 and L5 Lagrange points, points in a two-body system where the net force on a third mass equals zero [6].

Acknowledgments

I would like to thank my advisor and physics teacher, Dr. Mark Vondracek, whose continual guidance has made all this possible. I would also like to thank Dr. Ray Pierrehumbert and Mr. Jonathan Mitchell of the University of Chicago for their assistance with the energy budget model, and my father, Bill Kath, for helping me learn the mathematics and computer science that made this study a reality. I would finally like to thank my entire family for their infinite patience and support.

References

[1] Edwards, C. Henry and David E. Penney. Differential Equations and Boundary Value Problems. Upper Saddle River, NJ: Prentice Hall, 2000.

[2] Evans, J.C. Binaries and Star Clusters. 26 October 1998. Online, Internet at http://www.physics.gmu.edu/classinfo/astr103/CourseNotes/ECText/ch14_txt.htm#14.2 (7 September 2004).

[3] Haghighipour, Nader, Jocelyn Couetdic, Ferenc Varadi, and William B. Moore. "Stable 1:2 Resonant Periodic Orbits in Elliptic Three-Body Systems." The Astrophysical Journal 596 (2003 October 20): 1332-1340.

[4] Hartmann, Dennis L. Global Physical Climatology. San Diego: Academic Press, 1994.

[5] MATLAB 6.1. Natick, Mass: The MathWorks, Inc., 2001.

[6] McCuskey, S.W. Introduction to Celestial Mechanics. Reading, Mass.: Addison Wesley Publishing Company, Inc., 1963.

[7] Peterson, Ivars. Newton's Clock: Chaos in the Solar System. New York: W.H. Freeman and Company, 1993.

[8] Pierrehumbert, Raymond. "A First Course in Climate: Earth and Elsewhere." Course notes for Geosciences 232, University of Chicago. Online, Internet at http://geosci.uchicago.edu/ rtp1/geo232/Notes.pdf (7 September 2004).

[9] Pierrehumbert, Raymond, Professor of Geosciences at University of Chicago. Interview by the author of this paper, 7 September 2004, Evanston, Ill.

[10] Seydel, Rudiger. From Equilibrium to Chaos: Practical Bifurcation and Stability Analysis. New York: Elsevier, 1988.

[11] Sol Station. "Alpha Centauri 3." Online, Internet at http://www.solstation.com/stars/alp-cent3.htm (18 September 2004).

[12] Thorsett, S.E., Z. Arzoumanian, F. Camilo, and A.G. Lyne. "The Triple Pulsar System PSR B1620-26 in M4." The Astrophysical Journal 523 (1999 October 1): 763-770.